Minimum Spanning Trees – Handout (by Schalk-Willem Krüger)

Definitions

Spanning Tree: A spanning tree of a graph is a subgraph that contains all the vertices of the graph and some or all of the edges

Minimum Spanninng Tree: A minimum-weight tree in a weighted graph which contains all of the graph's vertices.

Application

MST's can be applied to problems like phone networks, computer networks and trail networks

Prim's algorithm

```
1. let T be a single vertex
2. while (T has fewer than n vertices) {
3.       find the smallest edge connecting a vertex not in the tree to
              a vertex in the tree
4.       add it to T
5. }
```

Prim's algorithm can be speeded up by using other data structures, for example a heap. Use a heap to remember, for each vertex, the smallest edge connecting T with that vertex.

Running time: $O(n^2)$. Using heap: $O(m + n \log n)$

Kruskal's algorithm

- Sort edges in order of increasing weight.
- Process edges in sort-order.
- For each edge, add it to the MST if it does not cause a cycle.

Pseudo-code:

```
1. Initialize MST to be empty;
2. Place each vertex in its own set;
3. Sort edges of G in increasing-order;
4. for each edge e = (u,v) in order
5.       if u and v are not in the same set
6.          Add e to MST;
7.          Compute the union of the two sets;
8.       endif
9. endfor
10.return MST
```

Running time: $O(m \log m)$